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The Global Equilibrium Real Interest Rate: Concepts, Estimates, and Challenges

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Abstract

Real interest rates have been persistently below historical norms over the past decade, leading economists and policymakers to view the equilibrium real interest rate as likely to be low for some time. Various definitions and approaches to estimating the equilibrium real interest rate are examined, including approaches based on the term-structure of interest rates and small macroeconomic models. The individual-country approaches common in the literature are extended to allow for global trend and cyclical factors. The analysis finds that global factors dominate the downward trend in the equilibrium interest rate across 13 advanced economies. A corollary of this finding is that the U.S. equilibrium rate may be substantially lower than estimated in U.S.-only studies. The analysis also highlights how the common global trend confounds empirical assessments of the determinants of movements in the equilibrium rate and the need to better integrate term-structure and macroeconomic approaches.

Keywords: World Interest Rate, Natural Rate, Global Factors

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1. Introduction

Economists and financial market participants have gained conviction that the level of real (inflation-adjusted) interest rates likely to prevail over the long run—*the equilibrium real interest rate*—has declined to a level below the norms experienced over most of the past half century. These developments are global in scope, and research on the equilibrium real interest rate has had significant impacts on monetary policy discussions at the European Central Bank (e.g., Constancio, 2016) and at the Federal Reserve (e.g., Powell, 2018).

The low level of the equilibrium real interest rate has been explored using tools from macroeconomics and finance. The connections—and the gaps—between the macroeconomic and financial approaches remain underdeveloped. This analysis considers the macroeconomic and financial approaches from a global perspective. Both approaches identify a substantial decline in the level of real interest rates likely to prevail over the long run owing to a common global factor. The global approach reveals a more substantial decline in the equilibrium real interest rate in the United States than revealed by the typical individual-country approach, reflecting the low level of interest rates among advanced economies. However, there are substantial differences in the level of the equilibrium rate estimated by different methods and uncertainty about the level is considerable. In addition, the drivers of the decline in the equilibrium rate are difficult to identify. The results point to a need for further integration of financial and macroeconomic approaches to understanding trends in interest rates.

Following recent research, the equilibrium real interest rate is defined herein as a long-run notion, consistent with the equilibration of savings and investment over periods beyond the business-cycle frequency. This definition more generally implies that the equilibrium real interest rate is the real interest rate consistent with macroeconomic balance over the long run, as highlighted in section 2. The approach to estimation of the equilibrium real interest rate introduced by Laubach and Williams (2003) builds on the intuition of macroeconomic balance, extracting the trend in the real interest rate consistent with economic activity equal to its long-run potential level.

The analysis herein focuses on econometric approaches to estimating the equilibrium interest rate and its determinants, as outlined in sections 3 and 4. One approach estimates the equilibrium real interest rate using information from the term structure of interest rates (as in, for example, Bomfim, 2001; Bauer and Rudebusch, 2017; Christensen and Rudebusch, 2017; and Del Negro et al, 2019). The macroeconomic approach uses information from the relationship between output and interest rates to estimate the equilibrium real interest rate (as in, for example, Laubach and Williams, 2003; Clark and Kozicki, 2005; Messonier and Renne, 2007; Lubik and Matthes, 2015; Laubach and Williams, 2016; Holston et al, 2017; Johannsen and Mertens, 2018; Brand et al, 2018; Brand and Mazelis, 2019; Fries et al, 2019; Bielecki et al, 2019, Jorda and Taylor, 2019; Lewis and Vazquez-Grande, 2019; and Kiley, forthcoming).

Most earlier work proceeds on an individual-country basis—even multicountry studies, such as Holston et al (2017) and Jorda and Taylor (2019)—estimate the equilibrium rate at an individual country level and only consider comovement subsequently. I estimate the equilibrium real interest from a global perspective—that is, the models herein explicitly incorporate global developments in estimation of trends for individual countries, rather than estimating trends for individual countries and then extracting common trends. Del Negro et al (2019) also explicitly models a global trend in an approach using term structure data; these authors do not consider the approach based on the comovement of output and interest rates.

The global approach yields important insights. Across 13 advanced economies, the common world component explains the overwhelming majority of the decline in the equilibrium real interest rate. Moreover, consideration of global developments results is notably lower estimates of the equilibrium real interest rate in the United States than estimates obtained via the more common, single-country approach. Finally, the dominance of global factors suggest that research on the determinants of the equilibrium real interest rate should focus on such factors to a larger degree, as in Rachel and Smith (2017), Del Negro et al (2019), Obstfeld (2019), and Rachel and Summers (2019), as highlighted in section 5.

Before turning to the technical analysis, the practical relevance of the decline in the equilibrium real interest rate for economic policy is considerable. With regard to monetary policy, a low level of the equilibrium real interest rate implies that economic stabilization is impeded by the effective lower bound on nominal interest rates and may point toward adjustments in policy strategy to mitigate such adverse effects (e.g., Kiley and Roberts, 2017; Bernanke, Kiley, and Roberts, 2019; and Gali et al, 2019). At the same time, uncertainty about the equilibrium real interest rate implies it may be difficult to find the optimal, or even a very good, approach that relies on an understanding of the equilibrium rate’s level (e.g., Orphanides and Williams, 2002; Giammarioli and Valla, 2004; and Weber et al, 2007; Bielecki et al, 2019). Beyond short-term interest rate policy, shifts in the equilibrium rate affect the term structure and assessments of term premiums, and hence may influence assessments of the efficacy of quantitative easing. For example, analyses of the effects of quantitative easing on term premiums may be sensitive to how models of term premiums account for changes in the equilibrium rate (e.g., has quantitative easing since 2008 persistently depressed bond yields?).

Fiscal policy discussions should also account for the equilibrium real interest rate. A low equilibrium real interest rate alters the sustainability of alternative government debt paths and hence raises questions regarding the appropriate stance of fiscal policy, especially given potential constraints on monetary policy from the effective lower bound (e.g., Furman, 2016). The implications of a low equilibrium real interest rate for fiscal policy have attracted less research attention as of this writing. Rachel and Summers (2019) point to the potential for expansionary fiscal policy to raise the equilibrium real interest rate. Blanchard (2019) examines the implications of a low equilibrium real interest rate for debt sustainability.

The decline in the equilibrium real interest rate also has important implications for financial stability. Persistently low interest rates may create financial fragility by straining financial-sector profitability and by incentivizing leverage (e.g., Borio and Disyatat, 2014; Claessens et al, 2016). In addition, a core area in financial-stability assessments involves assessments of valuation pressures and risk-taking (e.g., Adrian et al, 2015; Aikman et al, 2017; and Adrian et al, 2019). The possibility that the decline in the equilibrium real interest rate reflects an increased premium on safety (e.g., Del Negro et al, 2017; Caballero et al, 2017) points to a decline in expected returns on government bonds and a smaller, or no, decline in expected returns on risky assets. All of these effects make assessments of risk-taking and valuations difficult.

The next section discusses the evolution of real interest rates over the past half century and defines the equilibrium real interest rate. In light of this definition, section 3 reviews approaches to estimating the equilibrium real interest rate and presents estimates. Section 4 reviews possible explanations for the decline and presents some new evidence. Section 5 highlights key open challenges.

2. Developments and Definitions

2.1 Real interest rates over the past half century

To set the stage, figure 1 provides a snapshot of global interest-rate developments, focusing on 13 advanced economies: Australia, Canada, Denmark, France, Germany, Italy, Japan, Korea, the Netherlands, Spain, Switzerland, the United States, and the United Kingdom. Real ex post short-term and long-term interest rates (constructed by subtracting consumer price inflation realized over the previous three years from a 3-month interbank (or related) interest rate or 10-year government bond yield) have hovered in the neighborhood of zero percent over the decade to 2019, substantially below the levels that prevailed over the previous three decades (panel A).[1]

{INSERT FIGURE 1 HERE}

Moreover, policymakers and market participants expect low real interest rates to persist. For example, participants in the Federal Open Market Committee, the monetary-policymaking body of the U.S. Federal Reserve System, project that the long-run value of the real federal funds rate will equal about ½ percent (panel B). In addition, forward real interest rates, as implied by inflation-protected government bonds in the United States and Germany, point to low, and even negative, real interest rates lasting beyond the next five years (panel C).

2.2 The equilibrium real interest rate (and the natural rate): Long vs. short run

To understand the low-frequency movements in real interest rates documented in figure 1, researchers have focused on the equilibrium real interest rate. Three definitions are common. The equilibrium real interest rate is alternatively defined as

[1] The data appendix, available in the working paper version of this article on the Federal Reserve's website, provides details on data sources and the specific series used.

- The level to which real interest rates will converge in the long run absent shocks (e.g., Rachel and Smith, 2017);
- The level of the real interest rate consistent with long-run equilibration of savings and investment (e.g.,, Obstfeld, 2019);
- The level of the real interest rate consistent with output equal to its long-run potential (e.g., Laubach and Williams, 2003).

The equivalence of these definitions stems from common notions of macroeconomic balance. For example, short-run shifts in savings or investment demand (owing to, for example, changes in consumer or business confidence) lead to transitory movements in real interest rates—movements that fade over time and are not associated with the drivers of lower-frequency movements in interest rates. In addition, these types of shifts drive business cycle movements in economic activity, pushing economic activity away from long-run potential. As a result, the definitions involving long-run equilibration of savings and investment and output equal to potential are closely linked. Finally, the first definition's focus on trend similarly abstracts from short-run considerations.

Despite their similarities, the alternative definitions are useful, as they have led researchers to develop different approaches to the estimation of the equilibrium real interest rate. The focus on the level to which real interest rates converge over the long run has led researchers to use techniques to estimate the equilibrium rate via simple trends or common long-run trends among interest rates (e.g., Del Negro et al, 2019). The definition involving the level of interest rates consistent with output at potential underlies the trend extraction model that dominates this literature, following Laubach and Williams (2003). And the approach focusing on equilibration of savings and investment drives analyses that use calibrated general equilibrium models to assess trends in the equilibrium real interest rate (e.g., Krueger and Ludwig, 2007; Gagnon et al, 2016; Rachel and Summers, 2019).

The focus on the long run in discussions of the equilibrium rate distinguishes this concept from the Wicksellian notion of the natural rate of interest, which is typically defined as the level of the real interest rate that would equilibrate output to potential (and/or achieve price stability) in the short run (e.g., Woodford, 2011). This concept fluctuates considerably in the short run, in response to business cycle shocks, and has been estimated in dynamic-stochastic-general-equilibrium (DSGE) models (e.g., Neiss and Nelson, 2008; and Edge et al, 2008). Kiley (2013) and Del Negro et al (2017) discuss differences between equilibrium and natural rates in DSGE models.

3. Estimation Approaches and Results

The approaches to estimating the equilibrium real interest rate vary somewhat, but each relies on the close connection between long-run equilibrium and reversion to trend over the long run. In essence, the various approaches span from simple trend extraction to trend extraction informed by the economic mechanisms that inform researchers understanding of real interest rate dynamics. Our discussion defers consideration of calibration approaches to section 4.

3.1 Trend extraction

The most direct approach to estimating the equilibrium real interest rate, or at least to informing assessments of a reasonable range for the level and variation in the equilibrium real interest rate, involve simple trend measures. For example, Hamilton et al (2016), Lunsford and Hamilton (2019), and Del Negro et al (2019) consider moving averages over long time spans (e.g., 10 years). The fact that estimation of an underlying trend requires a moving average of such a long time period—combined with data availability typically spanning several decades to perhaps a bit more than 100 years—underscores how the data will have trouble informing assessments of the degree of variation in the equilibrium real interest rate (Lunsford and Hamilton, 2019; and Kiley, forthcoming). For example, even the long macrohistory database of Jorda, Schularick, and Taylor (201x) contains only 14 non-overlapping decade observations.

The detrending approach followed herein involves an unobserved components model in which the short-term real interest rate fluctuates around an unobserved, random-walk trend—similar to the approach to estimation of an inflation trend in Stock and Watson (2007) and to the approach to estimation of the equilibrium real interest rate used in Fiorentini et al (2018). Specifically, the real short-term interest rate, $r^s(t)$, follows

Equation 1

$$\Delta r^s(t) = \rho \Delta r^s(t-1) + \lambda\big(r^*(t-1) - r^s(t-1)\big) + e(t),$$

Equation 2

$$r^*(t) = r^*(t-1) + v(t),$$

where $r^*(t)$ is the equilibrium real interest rate, $e(t)$ and $v(t)$ are independent and identically distributed (i.i.d.) shocks, and ρ and λ are parameters to be estimated. In this application, the parameters are estimated via Bayesian methods. A technical appendix to the working paper provides estimation details. Important aspects of the Bayesian approach are relatively loose priors on all parameters except the standard deviation of shocks to the trend $(v(t))$. Because real interest rates have varied relatively little over the past half century and previous work has found that estimation of the variance of the trend can be challenging (e.g., Del Negro et al, 2019 and Kiley, forthcoming), the prior distribution of the standard error of $v(t)$ is assumed to be an

Inverted Gamma distribution with a mean of 0.25 percent (at an annual rate) and a standard deviation of 0.25 percent.[2]

3.2 Common factor among interest rates and yield-curve models

The simple trend approach abstracts from relationships among economic variables. Approaches that use the information from the yield curve exploit the connections implied by term structure models. A parsimonious and flexible model of the yield on a N-period (real) bond, $r^N(t)$, is

Equation 3

$$r^N(t) = E(t)\left\{\sum_{j=0}^{N-1} r^s(t+j)\right\} + \Xi(t),$$

Equation 4

$$\Xi(t) = A(L)X(t) + v(t),$$

where $E(t)$ is the expectations operator conditional on period t information. The first term in equation 3 captures the expectations hypothesis of the term structure, and $\Xi(t)$ is a factor governing deviations from the expectations hypothesis. Such deviations could be based on equilibrium models of the term structure that impose no-arbitrage restrictions (as in Kim and Wright, 2005, and as discussed in Gurkaynak and Wright, 2012) or on simple time-series models. The central implications of this approach for estimation of the equilibrium real interest rate is that the short-term real rate $r^s(t)$ and long-term real rates $r^N(t)$ (potentially for multiple N) share a common trend according to equation 3 so long as $\Xi(t)$ is stationary.

In the related literature, authors follow different approaches. For example, Christensen and Rudebusch impose the absence of arbitrage and estimate a traditional term structure model. In

[2] Kiley (forthcoming) discusses the challenge associated with estimating the variance of the trend innovation and how this relates to the "pile up" problem associated with maximum-likelihood estimation of the model of Laubach and Williams (2003).

contrast, Del Negro et al (2019) simple employ a common-trends model for short- and long-term real interest rates. Importantly, the simpler common trends approach is an unrestricted version of the no arbitrage approach (in cases where the vector of factors is the same across models, and as emphasized in, for example, Cochrane, 2007).

For this reason and in the interest of simplicity, our term-structure based approach models the 3-month real interest rate $r^s(t)$ and the 10-year real interest rate $r^N(t)$ using the common-trends specification. In contrast to the simple trend approach, the model jointly considers developments across all 13 advanced economies. Denoting the 26x1 vector of real short- and long-term interest rates by $\begin{bmatrix} \Delta r^s(t) \\ \Delta r^N(t) \end{bmatrix}$, the model is given by

Equation 5

$$\begin{bmatrix} \Delta r^s(t) \\ \Delta r^N(t) \end{bmatrix} = P \begin{bmatrix} \Delta r^s(t-1) \\ \Delta r^N(t-1) \end{bmatrix} + \Lambda \left(SR^*(t-1) - \begin{bmatrix} r^s(t) \\ r^N(t) \end{bmatrix} \right) + E(t) + D \begin{bmatrix} c^s(t) \\ c^N(t) \end{bmatrix}.$$

In equation 5, the equilibrium real interest rate $R^*(t)$, a 13x1 vector, is the common trend in $\begin{bmatrix} \Delta r^s(t) \\ \Delta r^N(t) \end{bmatrix}$ for each of the 13 countries and the matrix S (of dimension (26x13) selects the appropriate country element of $R^*(t)$. $R^*(t)$ follows a random-walk with country-specific shocks $V(t)$ (of dimension 13x1) and a global shock $w(t)$,

Equation 6

$$R^*(t) = R^*(t-1) + V(t) + w(t).$$

In addition and returning to equation 5, E(t) is a 26x1 vector of i.i.d. shocks, and $\begin{bmatrix} c^s(t) \\ c^N(t) \end{bmatrix}$ is a 2x1 vector of transitory global shocks to short- and long-term interest rates, with the matrix D (of dimension 26x1) selecting the appropriate element for each variable. $\begin{bmatrix} c^s(t) \\ c^N(t) \end{bmatrix}$ follows an autoregressive process of order 2 (AR(2)). To reduce the number of parameters, the elements of P are restricted so that only own-country lags enter a country's equation, the elements of Λ are restricted so that error correction occurs only between the dependent variable and the appropriate country-specific trend, and the autoregressive processes for the common global cyclical components are univariate. As in the simple model above, estimation is via Bayesian methods with loose priors except on the standard errors of the innovations to $R^*(t)$.

Relative to the simple trend model, the term-structure model exploits information in both short- and long-term interest rates and includes global cyclical and trend factors.

3.3 Small macroeconomic models

The term structure model utilizes economic theory to a limited extent, as it is based on the notion that real short-term and long-term interest rates will share a common trend if term premiums are stationary. But this is a very limited reliance on theory relative to the definitions of the equilibrium real rate that motivate the analysis, which rely on notions of macroeconomic balance such as the long-run equilibration of savings and investment and the achievement of economic activity at potential levels.

The popular model of Laubach and Williams (2003) attempts to bridge the gap between motivation and application by exploiting the observed time series relationships between fluctuations of output around trend and interest rates.[3] The model is based on the notion that cyclical movements in output are tied to cyclical movements in the real interest rate relative to

[3] Laubach and Williams (2003) additionally include terms for inflation and, in some cases, the labor market; these additional terms aid identification of the business cycle component of output, but are not critical to the intuition on how the model identifies the equilibrium real interest rate.

the equilibrium interest rate—that is, the model is based on a standard IS curve from a textbook treatment (e.g., Woodford, 2011).

The specification herein links the cyclical component of output, $[y(t)]$ (a 13x1 vector), to its own lags, a country specific and global business cycle shock ($E(t)$ and $c(t)$, respectively), and the short-term real interest-rate gap $(R^*(t-j-1)-[r^s(t-j-1)])$

Equation 7

$$[y(t)] = \sum_{j=1}^{2} \rho_j [y(t-j)] + \theta \sum_{j=1}^{4} (R^*(t-j-1) - [r^s(t-j-1)]) + E(t) + Ic(t).$$

θ (>0) is the slope of the IS curve—with higher interest rates $r^s(t)$, relative to the equilibrium level, depressing output. As in the term-structure model, the dynamics across countries are correlated through the cyclical global shock ($c(t)$), and there are no additional cross-country (short-run) dependencies. In addition, the equilibrium real interest rate has country-specific and global components and follows equation 6. Finally, estimation is via Bayesian methods, with details provided in a technical appendix to the working-paper version.

Focusing on the intuition, equation 7 has two properties important for estimation of the equilibrium real interest rate. First, it has the standard trend-extraction property: If real interest rates are low, this likely reflects a mix of transitory shocks and declines in the equilibrium real interest rate. In addition, equation 7 implies that the combination of low real interest rates and output below potential is particularly indicative of a low equilibrium real interest rate, as low interest rates are otherwise expected to boost output relative to potential.

3.4 Results

The analysis focuses on the 13 countries for which real interest rate data were presented in figure 1: Australia, Canada, Denmark, France, Germany, Italy, Japan, Korea, the Netherlands, Spain, Switzerland, the United States, and the United Kingdom. As is apparent from figure 1, experience across these countries has varied considerably – with, for example, interest rates in many of these countries notably below U.S. levels in the years up to 2019. Details on sample period and estimation are provided in the technical appendix in the working paper. In sum, the sample roughly spans the mid-1960s to 2019, depending on country. For the small macroeconomic model approach, the cyclical component of output is obtained via the Hodrick and Prescott (1997) filter with a smoothing parameter of 140,000 (implying a smooth trend).

Figure 2 summarizes estimates of the equilibrium real interest rate for the last 25 years from each approach for the 13 countries. In each panel, the light-blue region represents the range of real short-term interest rates across the countries and the dark-blue region represents the interquartile range of estimates of the equilibrium real interest rate across countries.

{INSERT FIGURE 2 HERE}

Two takeaways are apparent. All approaches show a decline in the equilibrium real interest rate. In addition, the decline is more significant for the two approaches that take a global perspective and use the entire set of countries to inform the estimate of the equilibrium real interest rate. For example, the simple approach—which is applied country-by-country—yields a decline in the central tendency of estimates (the median) from about 2 percent in the mid-1990s to a bit below 1 percent in 2019. Both the term structure and simple macroeconomic model approach yield a decline in the median estimate of the equilibrium real interest rate from about 2 percent in the mid-1990s to about 0 percent in 2019.

Examination of the contribution of the global shocks to the evolution of the equilibrium real interest rate illustrates further the importance of a global perspective. Figure 3 presents the short-term real interest rate for the United States (solid blue line) along with the estimate of the equilibrium real interest rate (black-dashed line) and the contribution of the global shock to the equilibrium real interest rate (red-dashed-dotted line) for the past 25 years and for both the term-structure model and the small macroeconomic model. It is immediately apparent that the decline in the in the equilibrium real interest rate owes primarily to the global component.

{INSERT FIGURE 3 HERE}

Overall, the results on the importance of a global factor are intuitive and align closely with those of Del Negro et al (2019). At the same time, the prominence of the global factor raises questions about the country-by-country approach that has dominated both the early and recent literatures.

Figure 3 also highlights another central result in the analysis herein and in the related literature: Estimates of the equilibrium real interest rate can differ notably depending on the model used. The term-structure approach estimates that the equilibrium real interest rate in the United States equals about ½ percent in 2019. The small macroeconomic model estimates that the equilibrium real interest rate equals about -1 percent in 2019.

Figure 4 further demonstrates the importance of global factors by comparing the estimate from the small macroeconomic model of the equilibrium real interest rate for the United States using global data to an estimate from the same specification (equation 7) estimated using only U.S. data (and eliminating the global factors from the model and then re-estimating parameters). While general trends are similar and point to a decline in the equilibrium rate, the decline taking into account global factors is larger and the level as of 2019 is notably lower: The global model places the equilibrium real interest rate in the United States just below -1 percent, while the U.S.-

only approach places this concept close to 0 percent. This is intuitive in light of global developments—figure 1 shows the U.S. real short-term interest rate lies above the middle of the range witnessed across advanced economies since the mid-2010s, and hence the information for other countries suggests a lower equilibrium real interest rate.

{INSERT FIGURE 4 HERE}

More generally, a range of approaches suggests that the equilibrium real interest rate is likely notably below the norms of previous decades. These results are subject to considerable uncertainty, as discussed in section 5. The next section reviews some of the possible sources of the decline in the equilibrium real interest rate, as these factors bear importantly on the policy questions.

4. Determinants of the Equilibrium Real Interest Rate

4.1 Possible Determinants

An assessment of the factors contributing to decline in the equilibrium real interest rate requires a framework. While the approaches in the literature vary, most analyses use a simple model of the supply and demand for loanable funds. The supply of loanable funds is an upward-sloping function of the interest rate, reflecting the decisions of households, businesses, and governments; loosely speaking, higher returns to saving boost the supply of savings. The demand for loanable funds is driven by the same set of actors and is downward sloping, reflecting the response of borrowers to higher costs of funds. Figure 5 presents a simple figure summarizing equilibrium.

Before turning to evidence, a few points that are important in the literature can be gleaned from figure 5. First, empirical evidence appears to suggest that both the savings and investment schedule are relatively inelastic (i.e., steep) with respect to interest rates. This property implies that observed shifts in the volume of savings and investment may be modest relative to the shift in interest rates. Rachel and Smith (2017) emphasize how the observed shifts in volumes of savings and investment over recent decades have been modest, yet consistent with a potentially notable shift in the equilibrium real interest rate.

{INSERT FIGURE 5 HERE}

A second observation relates to the relevant market for loanable funds and the associated implications for empirical work. Many empirical exercises examine relationships between the determinants of savings and investment demand and trends in real interest rates at an individual country level (e.g., Lunsford and West, 2019) or using a panel of countries (e.g., Hamilton et al, 2016; Holston et al, 2017; Ferrero et al, 2017; and Aksoy et al, 2019). But investment funds flow across borders and these flows have been viewed as key factors contributing to global trends in interest rates. For example, Bernanke (2005) introduced the idea that a global savings glut was an important factor contributing to low (long-term) interest rates in the mid-2000s. Caballero et al (2017), summarizing related research by these authors and others, point to increased savings in emerging markets and a related need for safe assets produced in advanced economies. These observations lead several analyses and discussions to focus on global factors (e.g., Rachel and Smith, 2017; Obstfeld, 2019; and Rachel and Summers, 2019). As our analysis below will make clear, it is empirically very challenging to discern the impact of country-specific and global factors, reflecting the high correlation between demographic and economic growth trends across advanced economies.

More generally, savings and investment depend on a large set of factors and hence the potential determinants of shifts in the equilibrium real interest rate are large. Table 1 summarizes key determinants emphasized in the literature, building on discussions in Rachel and Smith (2017), Lunsford and West (2019), and Rachel and Summers (2019)—each of which is based on overlapping-generation models of long-run economic growth as in Gertler (1999). To be clear, each of the factors discussed is likely to affect both savings and investment, and the table is meant to highlight whether forces are likely to shift primarily investment or savings. Starting first with factors influencing investment, slower growth, associated with either lower trend productivity growth or lower growth in the labor force, decreases required increases in the capital stock and consequently lower investment demand and interest rates.

{INSERT TABLE 1 HERE}

Turning to savings, a range of social and policy factors are prominent in the literature. Focusing first on demographic factors, the dependency ratio (the fraction of the population below age 15 and above age 64) is likely inversely related to savings: A higher fraction of the population outside working age, and still consuming, should be associated with a lower saving rate. An increase in life expectancy, holding constant working years, is likely to raise aggregate saving as the working-age population boosts saving in anticipation of a longer retirement. Note that this is a transition and partial effect: An increase in life expectancy, holding constant years of work, eventually boosts the dependency ratio, which lowers savings. These effects occur over long periods and imply complex dynamics (e.g., Krueger and Ludwig, 2007; Gagnon et al, 2017). Outside demographics, government saving is an important determinant of national and global saving (e.g., Elmendorf and Mankiw, 1999). Finally, increased inequality is likely to boost savings and lower interest rates, as the propensity to consume of the rich is lower than that for other segments of the population (e.g., Dynan et al, 2004).

The bottom rows of table 1 focus of factors that may affect the demand for saving in safe investment vehicles. This contrasts with the top two-thirds of the table, which focused on overall savings and investment—and hence discusses the effects on interest rates, the marginal product of capital, and returns on risky assets. For example, an increase in fundamental risk should depress the risk-free rate in a standard asset-pricing model, without necessarily having similar effects on the marginal product of capital (e.g., Kiley, 2004). The demand-for-safe-assets mechanisms in Caballero et al (2017) should disproportionately affect safe interest rates, resulting in wider spreads of required returns on risky assets over safe assets. Other models have similar implications based on alternative drivers (e.g., Del Negro et al, 2017). These mechanisms are important to keep in mind, as research has emphasized that the decline in real interest rates over recent decades has not been accompanied by declines in measures of the return to productive capital (e.g., Farhi and Gourio, 2018), casting doubt on mechanisms in which movements in real interest rates and the return to capital are tightly linked.

4.2 Reduced-form regressions

Our empirical analysis of determinants follows recent analyses that relate measures of trends in real interest rates to some of the factors emphasized in the literature. The analysis considers only a few potential determinants, for two simple reasons. First, the underlying trends in real interest rates largely consist of a downward movement since the mid-1980s (figure 1). Many of the determinants similarly contain similar trends—with, for example, economic growth generally decelerating (on a trend basis) since that time and life expectancies generally increasing since the 1960s. These developments result in a high degree of correlation across determinants, making reduced-form empirical assessments challenging. In addition, data for an extended period on some potential determinants is not available.[4]

Following previous cross-country analyses, the empirical approach focuses on demographic factors and underlying growth trends. For demographic factors, the analysis considers the dependency ratio and average life expectancy at birth, both from the United Nations. For underlying growth trends, the five-year moving average of GDP growth is used. While other analyses have taken more complicated approaches, the equation herein relates the five-year moving average of the short-term real interest rate within a country to the determinants via a simple fixed-effects panel regression. In addition to the country-specific variables, a specification including the population-weighted averages of the independent variables, in addition to their country-specific values, is also considered. Table 2 reports results. Several results stand out. None of the potential determinants is significantly different from zero when considered in isolation (columns 1 to 3, i.e., each 90-percent confidence interval contains 0). Joint consideration of the dependency ratio, life expectancy, and underlying growth finds statistically significant relationships between the underlying real short-term interest rate, the dependency ratio, and life expectancy (column 4). Life expectancy has the expected negative sign (as a longer life expectancy is expected to boost saving for retirement and thereby lower interest rates). However, the dependency ratio has "the wrong sign". Note that Ferrero et al (2017) and Aksoy et al (2019) find a similar correlation, and Aksoy et al (2019) present a more complicated model in which a higher dependency ratio depresses trend innovation and growth

[4] For example, the World Inequality Database (https://wid.world/) covers the countries we analyze, but basic inequality measures are missing for many countries before recent decades.

and thereby depresses the real interest rate. Finally, adding the global averages (column 5) results in most variables being insignificant.

{INSERT TABLE 2 HERE}

This result is not surprising but highlights an important point. As emphasized in figure 3, most of the decline in the equilibrium real interest rate appears to owe to a common global factor that has fallen slowly—like a trend—over time. Many potential determinants of the equilibrium real interest rate, such as life expectancy, underlying growth, and inequality, similarly have exhibited high degrees of comovement dominated by a low frequency component over the past half century. As a result, empirical assessments are plagued by multicolliinearity. This is illustrated in figure 6, which plots the global average for life expectancy along with country experiences in the top panel and the global average for five-year GDP growth along with country experiences in the bottom panel. The similarity in patterns in the variables across time and countries is striking.

{INSERT FIGURE 6 HERE}

In addition to the econometric challenges associated with collinearity, the regression or correlation approach, while common, has conceptual issues. Most importantly, the steady-state or long-run relationships highlighted in table 1 can result in complicated dynamics that play out over decades, reflecting life-cycle effects associated with demographic transitions and the slow movement in the capital stock in response to persistent shifts in savings and investment. As a result, research has considered implications from calibrated general equilibrium models.

4.3 Calibrated general equilibrium models

The most common factors considered in calibrated general equilibrium models are demographic factors such as shifts in the dependency ratio and life expectancy, modeled in overlapping –generations models of various degrees of complexity (e.g., Krueger and Ludwig, 2004; Carvalho et al, 2016, Gagnon et al, 2016; and Brzoza-Brzezina and Kolasa, 2018). In general, these models find substantial effects of demographic changes. For example, Krueger and Ludwig (2007), in an analysis that predates much of the recent literature documenting a decline in the equilibrium real interest rate, projected a decline in the equilibrium rate from early-2000s levels of just below 1 percentage point. More recently, Carvalho et al (2016) and Gagnon et al (2016) both suggest that the aging of the baby boom generation and increases in life expectancy contribute to a decline in the equilibrium real interest rate from the mid-1980s to the present of approximately 1-1/2 percentage points. This estimate is a substantial share of the overall decline estimated herein and in related studies.

A more comprehensive assessment of reported in Rachel and Summers (2019)—although their estimates reflect an assembly of results from different models. Overall, they find a 1.7 percentage point decline in the equilibrium real interest rate. Demographic factors account for a 1.8 percentage point decline, in the range of Carvalho et al (2016) and Gagnon et al (2016). Importantly, Rachel and Summers also consider the effects of a decline in trend productivity growth and increased inequality. The combination of slower trend productivity growth and increased inequality is estimated to lower the equilibrium real interest rate by another 2-1/2 percentage points. Increased government debt and higher government spending on the elderly, the combination of which is estimated to raise the equilibrium real interest rate by 4 percentage points, offset these drags on the equilibrium real interest rate. Overall, the key takeaway from

Rachel and Summers (2019) is that a model with relatively inelastic savings and investment schedules can imply large, and potentially offsetting, movements in real interest rates in response to changes in demographics, productivity, and government policies. Whether such large offsetting effects are a good description of the data merits further analysis.

In addition, it is noteworthy that the calibrated general equilibrium framework has focused on factors influencing savings and investment and hence capital accumulation. These mechanisms predict similar movements in real interest rates and the marginal (and under some conditions, average) product of capital. Measures of the average product of capital do not show signs of a strong downward trend (e.g., Farhi and Gourio, 2018). The divergence between safe interest rates and broader measures of the return to capital points to a widening of risk spreads associated with demand for safe assets, not shifts in savings and investment, as potential drivers of the equilibrium real interest rate (e.g., Del Negro et al, 2017).[5]

5. Open Questions and Challenges

5.1 The term structure and macroeconomic models

A striking feature of the estimates in section 3, and in the literature, is the separation between the approach using information from the term-structure and the macroeconomic approach. This separation is clearly inconsistent with the notion that financial and macroeconomic conditions are jointly determined. But it is consistent with the common approaches taken in both the term structure and macroeconomic literatures.

[5] Eggertsson et al (2019) suggest that increases in monopoly rents have accompanied the shifts in savings and explain the divergence between measures of the average product of capital and real interest rates.

The role of macroeconomic factors in shaping the term structure of interest rates is a component of ongoing research in term-structure modeling (e.g., Gurkaynak and Wright, 2012). Large macroeconomic models typically incorporate important effects of short- and long-term interest rates on economic activity (e.g., Boivin et al, 2010). Nonetheless, the types of models used to estimate the equilibrium real interest rate from a macroeconomic or term-structure perspective have largely proceeded independently.[6]

Progress toward ameliorating this situation depends on developments in (at least) two areas. First, the macroeconomic literature has made limited progress in developing parsimonious models in which long- and short-term interest rates affect economic activity (e.g., Kiley, 2014). Such progress is a precondition for joint determination of economic activity, short-term interest rates, and long-term interest rates—and hence a precondition for models that combine the macroeconomic and term-structure approaches.

Relatedly, our understanding of the term structure needs to account for the non-stationarities induced in nominal interest rates by trends in inflation (e.g., Stock and Watson, 2007) and the equilibrium real interest rate. Traditional term-structure models assume nominal, or at least real, interest rates are stationary (e.g., Kim and Wright, 2005), with only limited exceptions (e.g., Christensen and Rudebusch, 2017). In addition to facilitating integration of term-structure approaches to estimation of the equilibrium real interest rate, accounting for trends in inflation and the equilibrium real interest rate may shift understanding of term structure dynamics.

[6] Johannsen and Mertens (2018) use term structure information in a small macroeconomic model of the equilibrium real interest rate for the United States, but do not consider the effects of both long- and short-term interest rates on economic activity.

5.2 Uncertainty regarding the level and determinants of the equilibrium rate

There is a high degree of uncertainty surrounding the equilibrium rate. For example, our discussion of the determinants of the equilibrium real interest rate highlighted how short data samples, substantial collinearity, and the potential for a multitude of offsetting effects cloud understanding. Moreover, our presentation of estimates of the equilibrium real interest rate highlighted how different approaches can provide quite different estimates. Overall, uncertainty is high, and this uncertainty has led many to question how discussions of the equilibrium rate enter policy discussions (e.g., Orphanides and Williams, 2002; Weber et al, 2007; and Bielecki et al, 2019).

The fact that the equilibrium real interest rate is unobserved and evolves slowly implies that uncertainty regarding its value—especially its contemporaneous value—is an irreducible feature. This has important practical consequences, illustrated in figure 7. First, estimates of the equilibrium rate based on data up to a given period revise substantially as more data arrives. This is shown in the top panel, which plots the one-sided estimate of the equilibrium real interest rate from the global small macroeconomic model along with the estimate that uses all available data. (Note that the last observation of these two concepts is identical by definition.) It is clear that a researcher or policymaker operating in real-time would have perceived the equilibrium real interest rate to be substantially higher than it is currently estimated to be in, for example, the 2000s; this impression is confirmed by examination of Federal Reserve staff estimates of the equilibrium real interest rate reported in staff forecast materials for the mid-2000s. Indeed, this figure makes clear that the evidence for a decline in the equilibrium real interest rate within this model essentially appeared in the wake of the Global Financial Crisis, as the one-sided estimate places the equilibrium real interest rate near 3 percent in 2008. The magnitude of these revisions drives home the challenges for policy analysis raised in Weber et al (2007) and Bielecki et al (2019).

{INSERT FIGURE 7 HERE}

A corollary of this fact is that the credible interval surrounding estimates of the equilibrium real interest rate are wide, especially at the end of a data sample (as shown in the bottom panel). Even so, the top of the 90-percent credible interval is near 0 percent for the United States in 2019.

6. Summary

Policymakers and economists have taken on board the persistence of low real interest rates and widely view the equilibrium real interest rate as low by historical standards. A range of approaches to estimating the equilibrium real interest rate confirm a pronounced downward trend among advanced economies in the level of real short-term interest rates likely to prevail over the longer term. Several areas deserve further scrutiny.

First, the analysis herein extended the dominant country-by-country approach to estimation of the equilibrium real interest rate to a global perspective. A joint global model of the term structure or of the macroeconomic approach shows that global factors overwhelmingly drive the decline in the equilibrium real interest rate. This emphasizes that the source of the decline is likely to be global suggesting that the econometric analyses of the determinants of the equilibrium real interest rate that use panels of countries may need to consider alternative approaches. The global approach also suggests that the equilibrium real interest rate in the United States may be lower than appreciated, reflecting the low level of interest rates among advanced economies relative to U.S. levels.

In addition, the term structure and macroeconomic approaches to estimation of the equilibrium real interest rate have largely proceeded along parallel tracks, and it would be useful to see feedback among these workstreams. This requires models of the term structure to incorporate trends in interest rates. It also requires richer macroeconomic models of how the term structure of interest rates affects economic activity.

Finally, the equilibrium real interest rate is very difficult to estimate in real time. While all indications point to a level of the equilibrium real interest rate that is very low by historical standards in the late 2010s, researchers and policymakers should be prepared to revise their views—in whatever direction the data takes them—in coming years. A look at estimates of the equilibrium real interest rate using data through 2008—estimates that pointed to much higher estimates of the equilibrium rate than current data—illustrates the potential for large changes in assessments.

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Table 1: Factors Expected to Shift Aggregate Investment and Savings

Factors Primarily Shifting Aggregate Investment
Economic growth • Slower trend productivity growth decreases required increases in the capital stock and consequently lower investment demand and interest rates. • Slower growth in the labor force decreases required increases in the capital stock and consequently lower investment demand and interest rates.
Factors Primarily Shifting Aggregate Savings
Demographic factors • A higher fraction of the population outside working age—a higher dependency ratio—should be associated with a lower saving rate. • An increase in life expectancy, holding constant working years, is likely to raise aggregate saving as the working-age population boosts saving in anticipation of a longer retirement. **Fiscal policy** • Government saving is an important determinant of national and global saving and hence influences interest rates. **Inequality** • Finally, increased inequality is likely to boost savings and lower interest rates, as the propensity to consume of the rich is lower than that for other segments of the population.
Factors Primarily Shifting the Demand for Safe Assets
• An increase in fundamental risk should depress the risk-free rate in a standard asset-pricing model. • An increased demand-for-safe-assets from emerging-market economies. • Increased demand for safe assets driven by regulatory or other factors.

Table 2: Estimates of the Relationship Between

the Five-year Moving Average of Real Interest Rates and Determinants

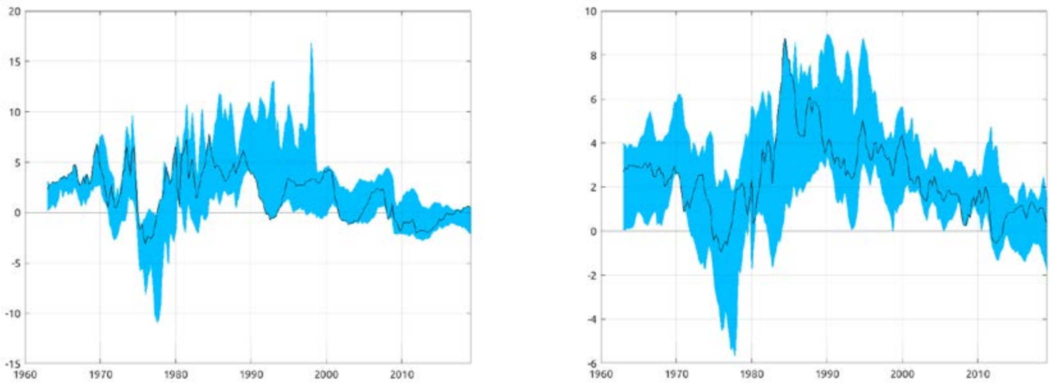
Across 13 Advanced Economies

Dependent (left-hand-side) variable: Five-year Moving Average of the Real Short-term Interest Rate within a Country					
Independent (right-hand-side) Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Dependency Ratio	-0.07 (-0.22,0.5)			-0.17 (-0.30,-0.07)	0.04 (-0.03,0.11)
Life Expectancy		-0.27 (-0.55,0.02)		-0.32 (-0.54,-0.05)	0.49 (-0.11,1.19)
GDP Growth			0.45 (-0.09,0.99)	0.38 (-0.17,0.96)	0.28 (-0.19,0.85)
Aggregate (population-weighted) Dependency Ratio					-0.31 (-1.87,0.85)
Aggregate (population-weighted) Life Expectancy					-0.61 (-1.10,-0.27)
Aggregate (population-weighted) GDP Growth					-1.09 (-2.12,-0.33)

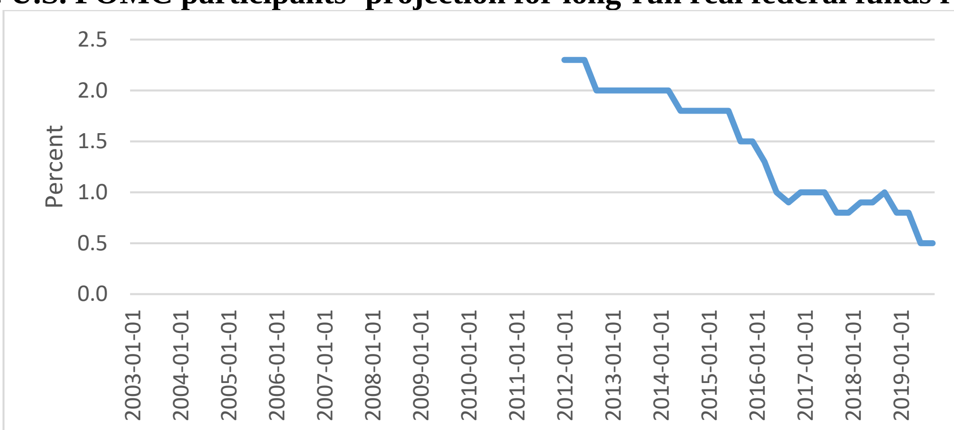
Note: Regressions include country-fixed effects.

Figure 1

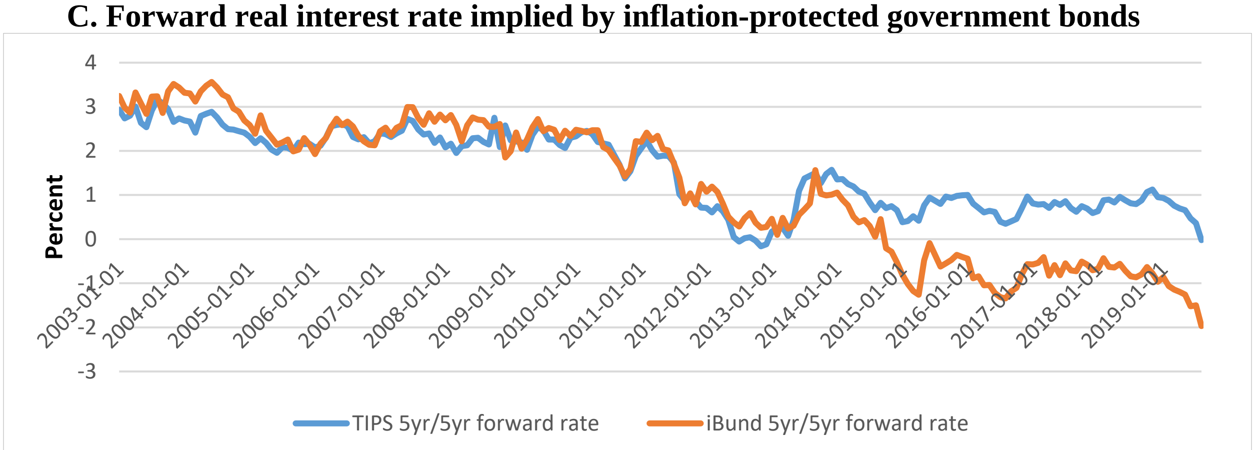
A. Ex post real short-term (left) and long-term (right) interest rates across 13 economies (Blue-shaded region: range; black solid line: U.S. data)



B. U.S. FOMC participants' projection for long-run real federal funds rate



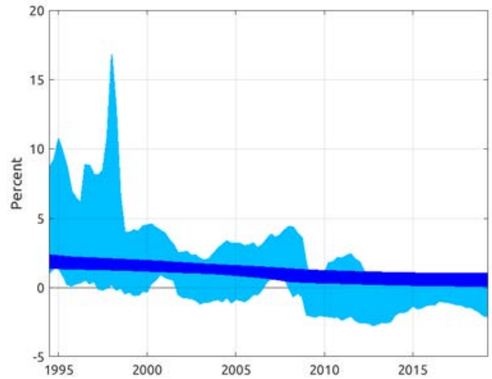
C. Forward real interest rate implied by inflation-protected government bonds



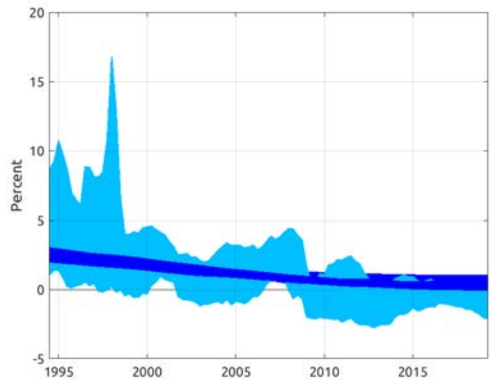
Source: Panel A—Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis; Panel B—Author's calculations and Federal Reserve Board, obtained from FRED at Federal Reserve Bank of St. Louis; Panel C—Author's calculations and U.S. Treasury and Deutsche Bundesbank.

Figure 2: Ex-post real short-term interest rates across 13 economies and the interquartile range of estimates of the equilibrium real interest rate (Light-blue: range for short-term real rate; Dark-blue: range for equilibrium rate)

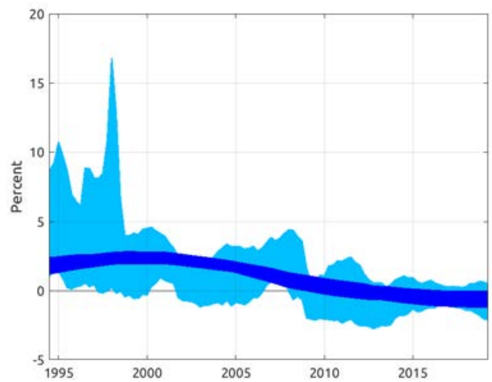
A. Simple-trend approach



B. Term-structure approach



C. Small-macroeconomic-model approach

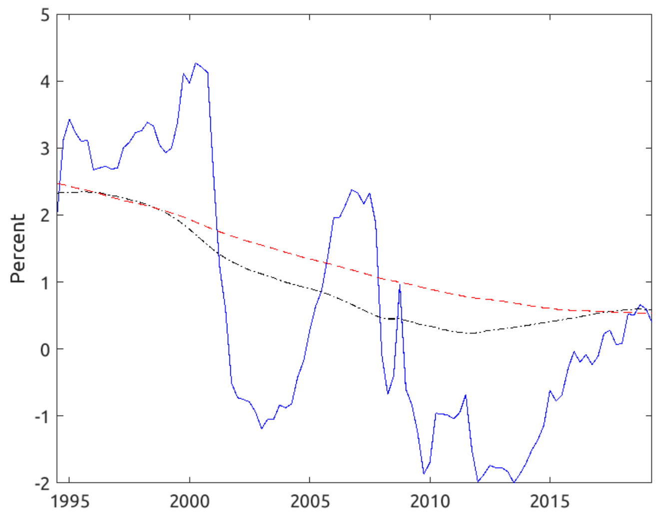


Source: Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis.

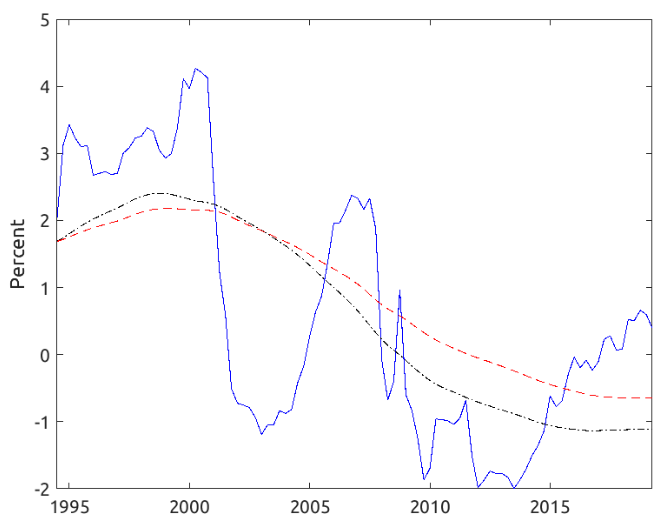
Figure 3: The ex-post real short-term interest rate in the United, the equilibrium real interest rate, and the contribution of the global shocks to the equilibrium real interest rate

Blue-solid: short-term real interest rate; Black-dashed: Equilibrium rate; Red-dashed-dotted: Contribution of global shocks)

A. Term-structure approach

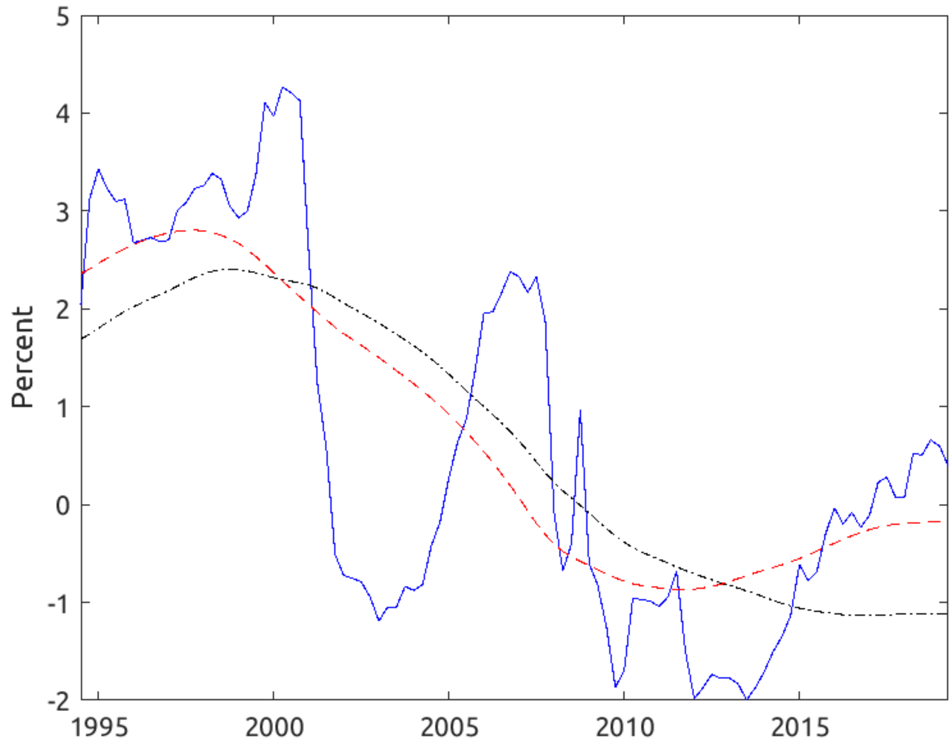


B. Small-macroeconomic-model approach



Source: Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis.

Figure 4: The ex-post real short-term interest rate in the United and the equilibrium real interest rate from global and US-only models (Blue-solid: short-term real interest rate; Black-dashed: Equilibrium rate—global model; Red-dashed-dotted: equilibrium rate—US-only model)



Source: Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis.

Figure 5: Saving and Investment Equilibrium

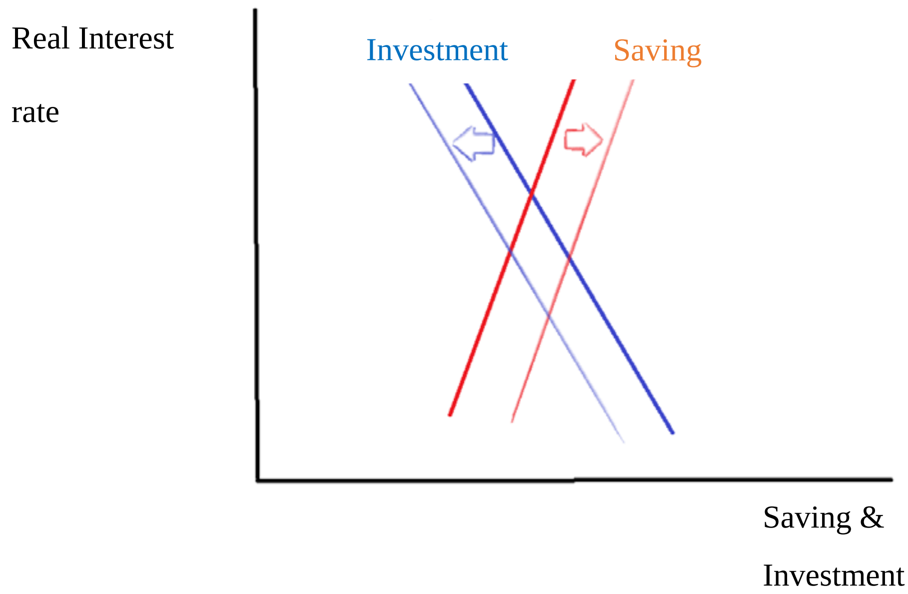
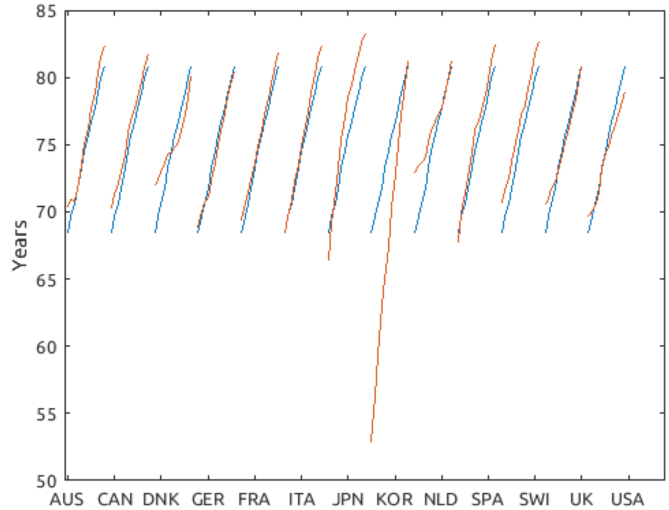
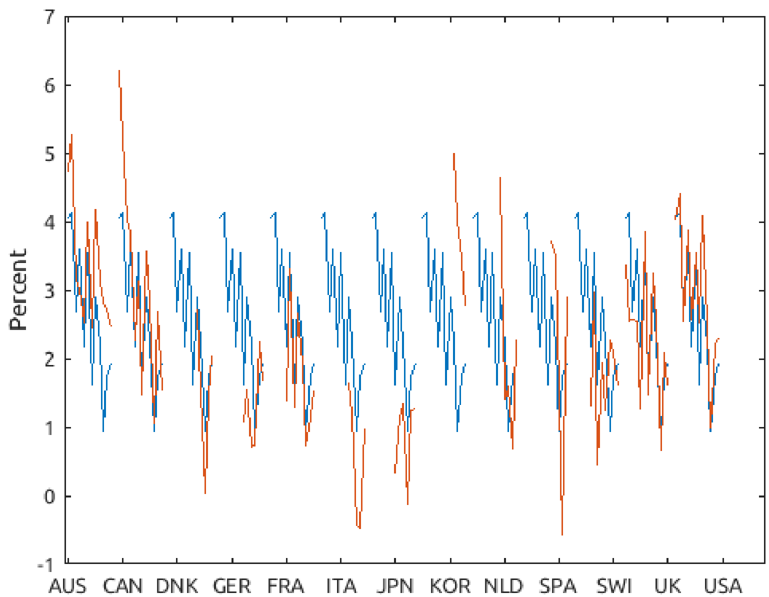


Figure 6: Trends in Life Expectancy and Economic Growth (1960-2015)

A. Life Expectancy-Global Average (Blue) and Country Data (Red)



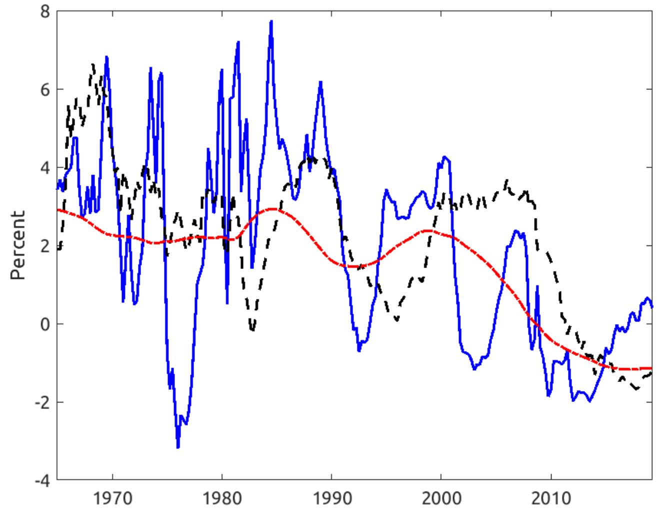
A. Five-year Real GDP Growth—Global Average (Blue) and Country Data (Red)



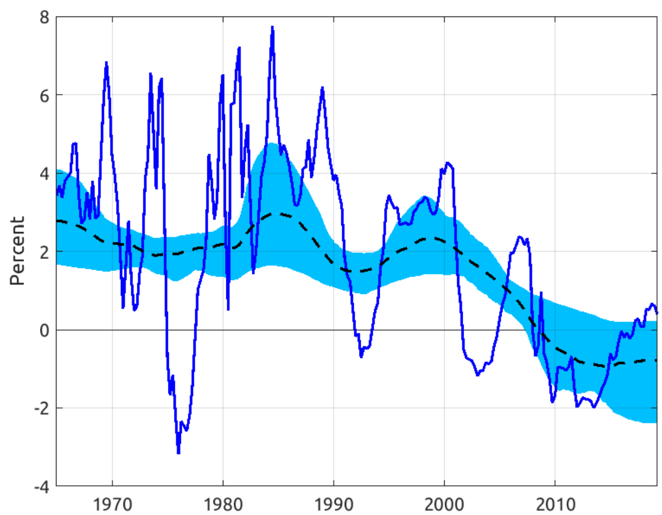
Source: Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis.

Figure 7: Uncertainty about the equilibrium real interest rate

Panel A. The ex-post real short-term interest rate in the United and the 1-sided and 2-sided estimates of equilibrium real interest rate from the global macroeconomic model (Blue-solid: short-term real interest rate; Black-dashed: Equilibrium rate—one-sided; Red-dashed-dotted: equilibrium rate—two-sided)



Panel B. The 2-sided estimates of equilibrium real interest rate from the global macroeconomic model and the 90-percent credible interval



Source: Author's calculations and OECD, obtained from FRED at Federal Reserve Bank of St. Louis.

Online Data appendix

The data on population, life expectancy, and dependency ratios are taken from the United Nations. As noted in the source note to figure 1, the data on yields on U.S. and German inflation-protected government securities are taken from the U.S. Treasury and Deutsche Bundesbank, respectively.

The data on nominal short- and long-term interest rates, consumer prices, and real GDP are from the OECD, downloaded from the FRED website maintained by the Federal Reserve Bank of St. Louis. The specific series are detailed below, including links to the website.

3-Month or 90-day Rates and Yields: Certificates of Deposit for Japan

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q3 1979 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Eurodollar Deposits for the United States

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for Australia

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1968 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for Canada

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for Germany

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for Italy

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q4 1978 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for Switzerland

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1974 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for the Netherlands

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1982 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Interbank Rates for the Republic of Korea

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1991 to Q3 2019 (2019-10-09)

3-Month or 90-day Rates and Yields: Treasury Securities for the United Kingdom

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q2 2017 (2018-02-26)

Constant Price Gross Domestic Product in Australia

Chained 2007-2008 Australian Dollars, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1960 to Q2 2019 (2019-09-11)

Consumer Price Index: All Items for Denmark

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1967 to Aug 2019 (2019-10-09)

Consumer Price Index: All Items for Korea

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Consumer Price Index: All Items for Netherlands

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Apr 1960 to Aug 2019 (2019-09-11)

Consumer Price Index: All Items for Spain

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index: All Items for Switzerland

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Consumer Price Index: All Items for the United States

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index of All Items in Australia

Index 2015=100, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q2 2019 (2019-08-12)

Consumer Price Index of All Items in Canada

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index of All Items in France

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Consumer Price Index of All Items in Germany

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index of All Items in Italy

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index of All Items in Japan

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Consumer Price Index of All Items in the United Kingdom

Index 2015=100, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Aug 2019 (2019-10-09)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Canada

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1961 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Denmark

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1995 to Q2 2019 (2019-10-09)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for France

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1980 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Germany

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1991 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Italy

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1996 to Q2 2019 (2019-10-10)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Japan

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1994 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Spain

Chained 2000 National Currency Units, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1995 to Q2 2019 (2019-10-09)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for Switzerland

Chained 2000 National Currency Units, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1980 to Q2 2019 (2019-10-09)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for the Netherlands

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1996 to Q2 2019 (2019-10-09)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for the Republic of Korea

National Currency, Quarterly, Seasonally Adjusted, Vintage: Current Q1 2000 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for the United Kingdom

Chained 2000 National Currency Units, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1960 to Q2 2019 (2019-09-11)

Gross Domestic Product by Expenditure in Constant Prices: Total Gross Domestic Product for the United States

Chained 2012 National Currency Units, Quarterly, Seasonally Adjusted, Vintage: Current Q1 1960 to Q2 2019 (2019-10-09)

Immediate Rates: Less than 24 Hours: Call Money/Interbank Rate for France

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q3 2019 (2019-10-09)

Immediate Rates: Less than 24 Hours: Call Money/Interbank Rate for Spain

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q2 1973 to Q3 2019 (2019-10-09)

Immediate Rates: Less than 24 Hours: Central Bank Rates for Denmark

Percent, Quarterly, Not Seasonally Adjusted, Vintage: Current Q1 1960 to Q3 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Australia

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jul 1969 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Canada

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Denmark

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1987 to Aug 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for France

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Germany

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Italy

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Mar 1991 to Aug 2019 (2019-09-11)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Japan

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1989 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Spain

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1980 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for Switzerland

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the Netherlands

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the Republic of Korea

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Oct 2000 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the United Kingdom

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the United States

Percent, Monthly, Not Seasonally Adjusted, Vintage: Current Jan 1960 to Sep 2019 (2019-10-09)

Online Estimation Appendix

Estimation is performed in Dynare (Adjemian et al, 2011). Replication codes are available on request. The remainder of this appendix gives some details on the priors used in estimation for each model.

Simple trend approach

Equation 8

$$\Delta r^s(t) = \rho \Delta r^s(t-1) + \lambda\big(r^*(t-1) - r^s(t-1)\big) + e(t),$$

Equation 9

$$r^*(t) = r^*(t-1) + v(t),$$

The prior for ρ is Normal with mean 0 and standard deviation of 2 (i.e., relatively uninformative). The prior for λ is Inverted Gamma with a mean of 0.0625 and a standard deviation of 0.125 (i.e., the prior implies modest error correction). The prior for the standard deviation of e(t) is Inverted Gamma with mean and standard deviation of 0.5; the prior for the standard deviation of v(t) is Inverted Gamma with mean and standard deviation of 0.25.

Term structure approach

Equation 10

$$\begin{bmatrix} \Delta r^{s}(t) \\ \Delta r^{N}(t) \end{bmatrix} = P \begin{bmatrix} \Delta r^{s}(t-1) \\ \Delta r^{N}(t-1) \end{bmatrix} + \Lambda \left(SR^{*}(t-1) - \begin{bmatrix} r^{s}(t) \\ r^{N}(t) \end{bmatrix} \right) + E(t) + D \begin{bmatrix} c^{s}(t) \\ c^{N}(t) \end{bmatrix}.$$

Equation 11

$$R^{*}(t) = R^{*}(t-1) + V(t) + w(t).$$

The prior for each element of P (a 4x4 matrix) is Normal with mean 0 and standard deviation of 2 (i.e., relatively uninformative). The prior for Λ imposes that it is diagonal and is Inverted Gamma with a mean of 0.0625 and a standard deviation of 0.125 (i.e., the prior implies modest error correction). The prior for the standard deviation of E(t) and e(t) is Inverted Gamma with mean and standard deviation of 0.5; the prior for the standard deviation of V(t) is Inverted Gamma with mean 0.125 and standard deviation of 0.25, and that for w(t) is Normal with mean 0.0625 and standard deviation 0.025.

Small macroeconomic model

Equation 12

$$[y(t)] = \sum_{j=1}^{2} \rho_j [y(t-j)] + \theta \sum_{j=1}^{4} (R^{*}(t-j-1) - [r^{s}(t-j-1)]) + E(t) + Ic(t).$$

The priors for equation 6 are the following in this model: The prior for the standard deviation of V(t) and w(t) are Inverted Gamma with mean 0.25 and standard deviation of 0.25.. In equation 7, the prior for the standard deviations of E(t) and the innovation to c(t) are Inverted Gamma with mean and standard deviation of 0.5. The priors for ρ_j are Normal with mean 0 and standard deviation 2; these same priors are used for the Ar(2) coefficients governing the world cycle, c(t). The priors for the elements of θ are Inverted Gamma with mean of 0.0625 and standard deviation of 0.25.